Clustering Algorithms for Non-Profiled Single-Execution Attacks on Exponentiations

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Motivation

- Single execution side-channel attacks on exponentiations
- Previous ones require profiling or manual tuning or use ad-hoc algorithms
- We describe how to use cluster classification algorithms instead

Reminder: Exponentiation Algorithms

- Exponentiations in asymm. crypto
  - Modular exponentiations in RSA
  - Elliptic curve scalar multiplications in ECC
- Popular algorithms:
  - Square-and-multiply-always (RSA) / double-and-add-always (ECC)
  - Montgomery ladder (RSA, ECC)
- Key features of exponentiation algorithms
  - Secret exponent processed bit/digit-wise in loop
  - Mostly timing-safe, hence, operation sequence uniform (against SPA)

Single-Execution Leakage

- Side-Channel Attackers only have **single observations** to exploit
  - Due to ephemeral exponent or e.g. blinding countermeasure

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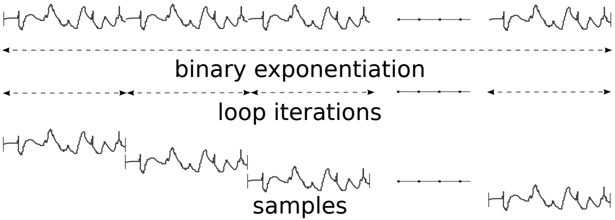
- **Certain amount of information about exponent bits** (binary alg.) is still leaking in most cases $\rightarrow$ **single-execution leakage** (adress-bit-related, localized leakage, ...

Exploiting Single-Execution Leakage



- Cut recorded exponentiation trace into samples
- Each corresponds to different secret bit (binary exp. alg.)
- **Attack basically means to find correct partition = Classification**

Exploiting Single-Execution Leakage
Previously and Strongly Related

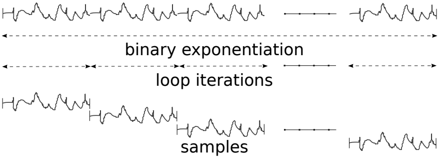
- **Template attacks**
  - **Require profiling** (difficult, think of e.g. blinding)
- **Cross-correlation-based attacks**
  - Requires **manually tuned thresholds**
  - Correlation disregards information (absolute values)
  - Some are based on heuristic power models
    (corr. coeff. makes more sense then)
- **Walter's Big Mac attack** from 2001
  - Ad hoc engineered algorithm

Our Proposal
Using Unsupervised Clustering for an Attack

- Use algorithms from the established research-field of '**Pattern classification**'
  - Those are already heavily researched in other applications
- We propose to use **unsupervised cluster classification algorithms**
  - Exploit single execution leakage of exponentiation algorithms

Our Proposal
Using Unsupervised Clustering for an Attack

- Reminder: In *profiled template attack*, cut-out samples are *classified* by *matching to templates*



- Clustering algorithms **classify** the cut-out samples **automatically without profiling or manual tuning**
  - Unknown if **0** or **1** bits, but easy try-out

- Success depends on available leakage of course

Unsupervised Cluster Classification Algorithms

- Unsupervised means no training data, no profiling
- Input a set of multi-dimensional samples/vectors e.g. cut-out trace-parts
- Algorithm estimates distributions
- Define free parameters of distribution (e.g. *two* cluster centers)
- Optimal algorithm depends on the distribution model (shape of clusters)

Unsupervised Cluster Classification Algorithms
K-Means

- Example algorithm:
  ***k*-means** algorithm for **unsupervised clustering**
  - Finds ***k*** cluster centers and corresponding classification
  - Distribution assumption - shape of clusters:
    - *k* equal Gaussian distributions
    - Independent values in samples (dimensions are independent)
    - Variance equal within clusters

Unsupervised Cluster Classification Algorithms
K-Means

- **Input:** Samples (cut-out trace parts) and number of clusters k
- Starts by choosing k **random** samples as initial cluster means
- Then iteratively:
  - Compute *Euclidean distance* from all samples to current k means
  - Classification: *Assign all samples to closest mean* $\rightarrow$ k classes
  - **Compute new means** of k classes from current classification
  - Repeat *until no change in class assignment*
- **Output:** k cluster means and classification
- Repeat with different starting points to prevent local maxima (best outcome based on sum-of-squared-error criterion selected)

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Practical Evaluation

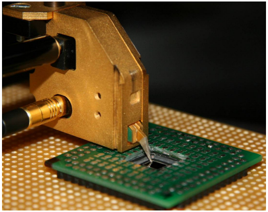
- Laboratory setup (FPGA-based , trigger output, synchronized clock) (Definitely not real world ;)
- Same setup as in our CT-RSA'12 paper:
  Template attacks exploiting location-based leakage

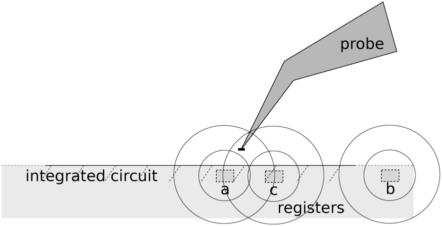
Practical Evaluation
FPGA DUT

- Straight-forward FPGA-based digital HW implementation:
  - Elliptic curve scalar multiplication ($Q = d \cdot P$) with affine input/output
  - López and Dahab Montgomery ladder 'exponentiation' algorithm, binary field $GF(2^{163})$, NIST parameters

Practical Evaluation
Location-Based Leakage

- High-resolution inductive near-field probe (**100 μm** resolution)
- Probe is closer to one of two registers
- Register access depends on current secret bit in loop





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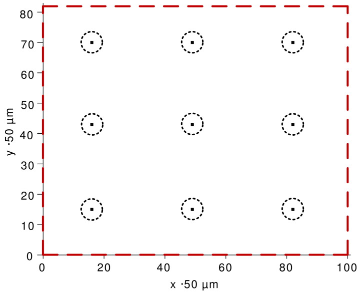
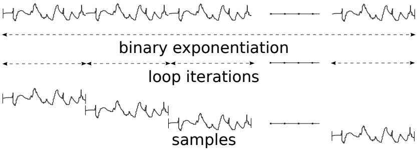
Practical Evaluation
Measurement Positions

- FPGA die surface
- Multiple measurement positions in geometric regular array (no profiling to find locations)



Practical Evaluation
Trace Example

- Reminder: Cutting a trace into samples



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Practical Evaluation
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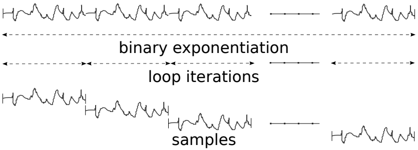
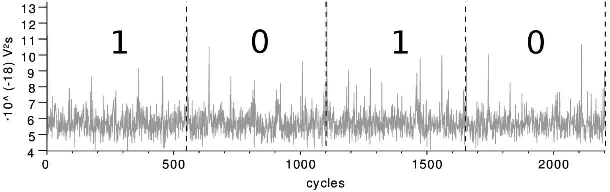


- Example from one measurement - 4 samples



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Practical Evaluation
Result from One Position

- Single measurement **after** clustering
  - Returns 2 sample means and corresp. classification

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  - **For visualization:**
    - Regard the samples/means as vectors in multi-dim. space
    - Draw line through to means
    - 1-D projection of samples on this line

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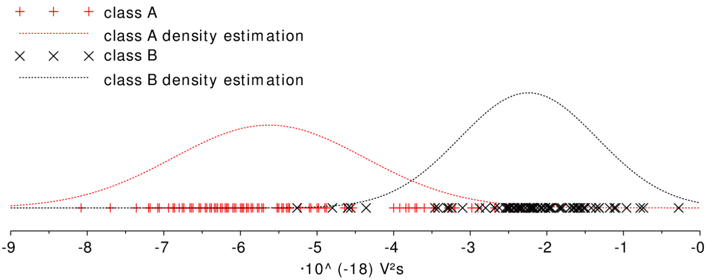

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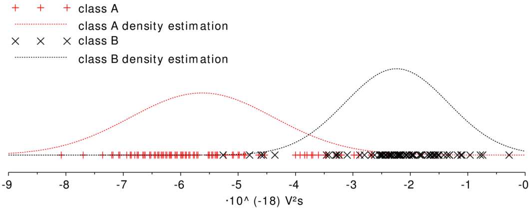
Practical Evaluation
How to Cope with Errors?

- Clustering algorithms allow to derive posterior probabilities for each sample describing likelihood of correct classification (basically low if close to separation plane)



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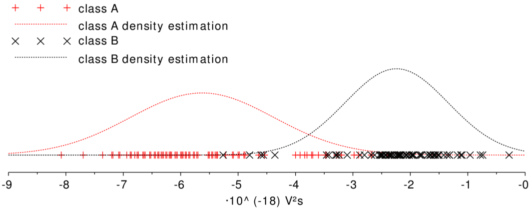

- Attacker may use this in a brute-force strategy:
  - Trial bits with low post. probabilities first
  - Repeat and increase number of trialed bits until correct exponent found

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Practical Evaluation
Results for All Positions

- Estimate remaining brute-force complexity **after** clustering attack

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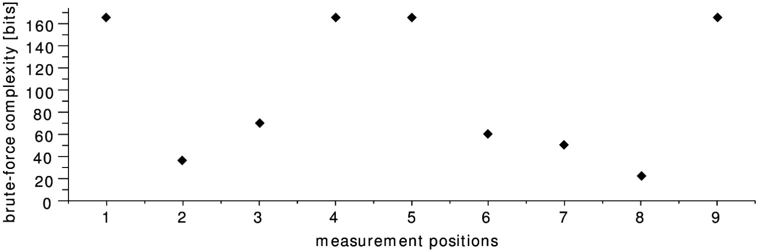
Results for All Positions

- Estimate remaining brute-force complexity **after** clustering attack
- All individual measurement positions:



- In **2 out of 9** cases, brute-force complexity is clearly feasible for attackers (only 2^{22} and 2^{37} trials)

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Practical Evaluation
Combining Simultaneous Measurements

- What if exploited leakage is insufficient?
- Repeating measurements is impossible because exponent changes

- Cluster analysis provides straight-forward possibility to combine (simultaneous) measurements:
  - Simply concatenate cut-out samples

Practical Evaluation
Improvement Through Combination

- Due to lack of mult. probes, meas. are repeated with const. inputs

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Improvement Through Combination

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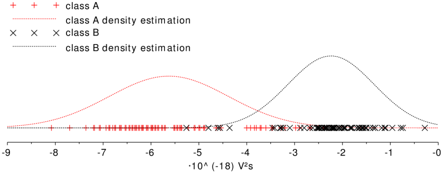
- **One** measurement (**after** clustering, 1-D projection): **Many Errors**



Practical Evaluation

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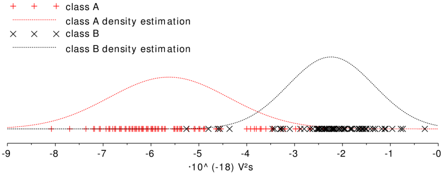
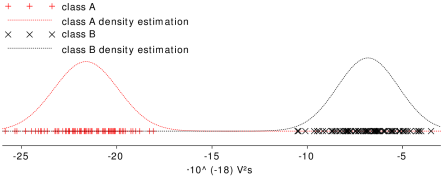


- **All** measurements (**after** clustering, 1-D projection): **No Errors**



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Countermeasures

- Exponent blinding or coordinate randomization do not help
- Reduce SNR of single-execution leakage as far as possible
- Address sources of specific single-execution leakage.
  E.g. Reduce location-based leakage using interleaved placement

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Conclusion

- **Non-profiled attack** against exponentiations
  - Well established clustering algorithms
  - No manual tuning
  - Can be generalized to any single-/multi-variate single execution leakage of exponentiation algorithms
  - Combination of measurements can improve attack
    $\rightarrow$ no need to find best positions

- In our opinion, this should make cross correlation-based single-execution attacks obsolete
- Clustering may also be interesting e.g. for SCA collision attacks

Thank You

Back-Up
K-Means

- Example: Graphical representation of 2-dimensional samples (not my data)
  - In this example: samples cluster around two means/centroids
  - This corresponds to binary exponentiation case
  - The segmentation can be found through unsupervised algorithms

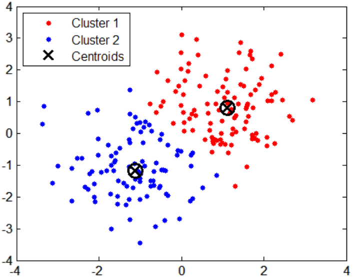


Figure: Source: `http://www.mathworks.de/de/help/stats/kmeans.html`

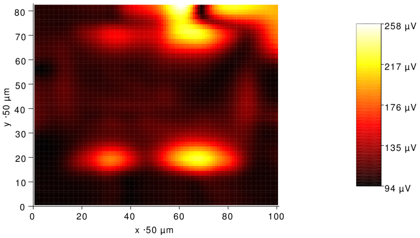
Back-Up
ECC Implementation

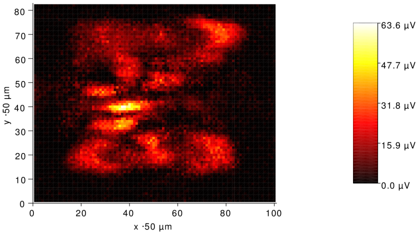
- Elliptic curve scalar multiplication ($Q = d \cdot P$)
- Binary field $GF(2^{163})$, NIST *Curve B-163* parameters
- López and Dahab Montgomery ladder 'exponentiation' algorithm
- Affine x- and y-coordinates as input and output

- Fulfills requirements for successfull attack
  - Bitwise processing of **163** bit scalar
  - Uniform operation sequence for each bit
  - Register usage **depends** on bits

Back-Up

Locations with High Leakage vs. High Amplitudes





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